Comment on ‘Quantum-Anti-Zeno Paradox’

Lajos Diósi*
Research Institute for Particle and Nuclear Physics

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Abstract

One can reduce the involved derivation of Balachandran and Roy of their ‘anti-Zeno’ effect [Phys.Rev.Lett. **84**, 4019 (2000)] to the derivation of standard Zeno-effect. The mechanism of what the authors call ‘anti-Zeno’ effect is a dynamic version of Zeno effect.

In a recent Letter [1], the well-known quantum-Zeno paradox is rederived. The authors obtain the correct effect in a complicated form. Furthermore, they suggest as if the quantum-Zeno effect were only related to the continuous measurement of a *constant* projection operator E. The Letter then exposes the issue of continuous measurement of the time-dependent projection $E_t = U_t E U_t^\dagger$, derives equations of state evolution, and interpretes the result as anti-Zeno paradox. The Letter claims that the time-dependent case has a new, even an opposite, physical consequence in comparison with the case of constant E. This is a misleading suggestion, I am afraid. The central purpose of my Comment is to correct the author's interpretation. I think the Zeno-mechanism itself is the same whether E is time-dependent or not. This is a most productive starting point. I derive the Letter's main result from the equations of common quantum-Zeno effect (where $E = const.$), using elementary transformations. The elegance of this method as well as the transparency of the resulting equation will be convincing.

The standard quantum-Zeno effect means in fact the constancy of the measured value (1 or 0) of the continuously measured projector E. The constancy of the quantum state follows from the constancy of E. If E depends on time in a smooth enough way, the standard Zeno effect survives perfectly: The value of a continuously measured time-dependent projector E_t remains constant (0 or 1), independently of the self-dynamics of the system.

*E-mail: diosi@rmki.kfki.hu

Correspondingly, Zeno effect means in its full generality that the quantum state ψ_t remains confined in the subspaces belonging to E_t for all time t during the continuous measurement. If E_t projects on a one-dimensional subspace then, provided the first measurement gives 1, i.e. $\psi_0\psi_0^\dagger = E_0$, the continuous measurement will be dragging the state ψ_t with itself and, independently of the Hamiltonian, the identity $\psi_t\psi_t^\dagger = E_t$ remains valid during the continuous measurement. This dragging-mechanism (known already to von Neumann and to others [2] as pointed out in Refs. [3,4]) is the veriest quantum-Zeno mechanism. It is misleading to interprete it as a new anti-Zeno effect. [The author's twist to the old metafore is a bit artificial. The watched kettle "boils" *iff* the path $\{E_t; t \in (0,T)\}$ reaches a boiling state. This is not necessarily sure for "most ways of watching".]

To support my viewpoint, I present an elementary derivation of the Letter's result, starting from the equations of standard Zeno effect. So, consider a system whose unitary evolution $\dot{\psi}_t = -iH_t\psi_t$ is governed by an arbitrarily given Hamiltonian (maybe of smooth time-dependence). Let the *constant* projector E be continuously measured and let us assume that the initial measured value is 1, *i.e.*, $\psi_0 = E\psi_0$. Then, the state will unitarily evolve with the effective Hamiltonian:

$$\dot{\psi}_t = -iEH_tE\psi_t \ . \tag{1}$$

As a consequence, the identity $E\psi_t = \psi_t$ will hold during the measurement; this is the standard Zeno effect. Eq. (1), which I take here granted, follows from common treatment of Zeno effect [5]. A smooth time-dependence of the Hamiltonian makes no difference [4]. Now we switch to the general case. Following the Letter, the smooth time-dependence of the measured projector E_t will be described by unitary rotations $E_t = U_tEU_t^\dagger$, we set $U_0 = 1$. Note that the rotations U_t are not unique while the projectors E_t are. It is trivial to inspect that this case goes back to the standard one in a frame rotated by U_t. Accordingly, I transform the state space as well as the observables: $\psi_t \to U_t^\dagger\psi_t$, $O_t \to U_t^\dagger O_tU_t$. Also the self-Hamiltonian transforms: $H_t \to U_t^\dagger H_tU_t + i\dot{U}_t^\dagger U_t$. Obviously, the projector to be observed becomes the constant E itself. We can therefore apply the Eq. (1) of standard quantum-Zeno. Then we return from the rotating frame to the original one, this equation takes the following ultimate form:

$$\dot{\psi}_t = -iE_tH_tE_t\psi_t + [\dot{E}_t, E_t]\psi_t \ , \tag{2}$$

leading to the general Zeno effect: $E_t\psi_t = \psi_t$. We see that the evolution (2) is always unitary since the effective Hamiltonian $E_tH_tE_t + i[\dot{E}_t, E_t]$ is hermitian. This Hamiltonian evolution applies to the general case when the state is described by density matrix. Another advantage of the above form is that, contrary to the Letter's Eq.(34), the unphysical unitary operators U_t do not at all appear in it. Only the projectors E_t do. The simplicity of Eq. (2) is remarkable as compared to the Letter's Eq. (34).

It is worthwile to add that the term 'anti-Zeno' had already been reserved for environmental *acceleration* of quantum transitions, see, e.g., Ref.[6] and references therein.

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[1] A.P. Balachandran and S.M. Roy, Phys. Rev. Lett. **84**, 4019 (2000).

[2] J. von Neumann, Matematische Grundlagen der Quantenmechanik (Springer, Berlin, 1932); Y. Aharonov and M. Vardi, Phys. Rev. **D21**, 2235 (1980).

[3] D. Giulini, E. Joos, C. Kiefer, J.Kupsch, I.-O. Stamatescu and H.D. Zeh: *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996).

[4] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1993).

[5] To verify the effective Hamiltonian EH_tE, consider the infinitesimal unitary evolution of $\psi_t = E\psi_t$ broken by the measurement of E at time $t+dt$ which amounts in $Ee^{-idtH_t}\psi_t$. This we can re-write as $e^{-idtEH_tE}\psi_t$.

[6] O.V. Prezhdo, Phys. Rev. Lett. **85**, 4413 (2000).